

Basics

Introduction

Be sure to revise the Electricity unit of G.C.E. (A/L) Physics.

Charge

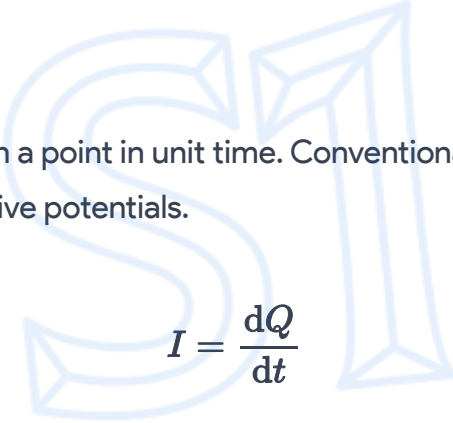
Quantized & conserved. Measured in Coulomb (C).

$$1 \text{ C} = \text{Charge of } 6.25 \times 10^{18} \text{ electrons}$$

Time invariant charge is denoted as Q . And time varying charge is denoted as q .

Current

Amount of charge flowing through a point in unit time. Conventional current (opposite to electron flow) flows from positive to negative potentials.


$$I = \frac{dQ}{dt}$$

Time invariant current (DC) is denoted as I . And time varying current (AC) is denoted as i .

Voltage

Voltage at a point is the work that must be done against the electric field to move a unit positive charge from infinity to that point.

1 volt is the potential difference between 2 points when 1 joule of energy is used to move 1 coulomb of charge from one point to the other.

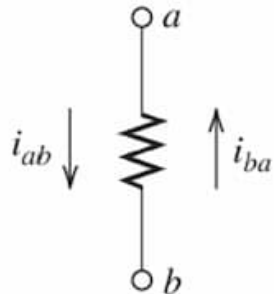
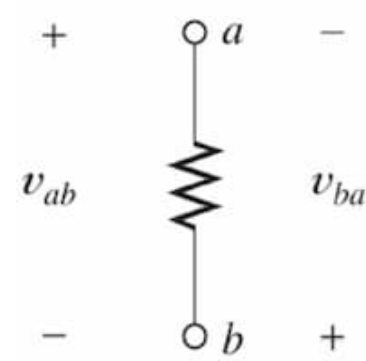
$$V = \frac{E}{Q}$$

Time invariant voltage is denoted as V . And time varying voltage is denoted as v .

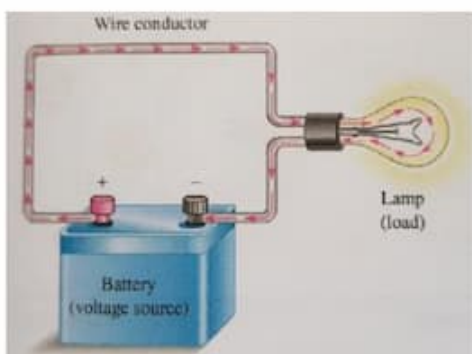
Voltage difference between 2 points is the work that must be done against the electric field to move a unit positive charge from one point to another.

$$V_{AB} = V_A - V_B$$

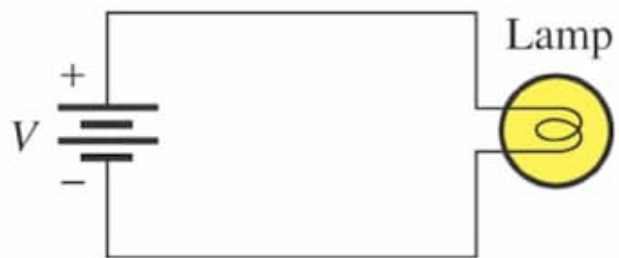
Double subscript notation

-	Current	Voltage
Double subscript		
Equation	$i_{ab} = -i_{ba}$	$v_{ab} = -v_{ba} = v_a - v_b$
Description	Current is flowing from point a to point b	Voltage is higher at point a and lower at point b

Electric Circuit



Pictorial diagram



Schematic diagram

Types of circuits

- Closed circuit - the electricity flows
- Open circuit - the electricity doesn't flow. current = 0 . ∞ resistance.
- Short circuit - very large current. 0 resistance.

Power

$$p = \frac{dw}{dt} = \frac{dw}{dq} \frac{dq}{dt} = vi$$

Total Work

$$w = \int_{t_0}^t p dt = \int_{t_0}^t vi dt$$

When v and i are constant

$$w = vi \int_{t_0}^t dt = vi(t - t_0)$$

Electrical Load

Something that consumes electrical energy.

Linear loads

Loads that have a linear relationship between the applied voltage and the current. Can be expressed using a combination of resistors, capacitors and inductors only.

Note

If a AC sinusoidal voltage is applied across a load, current through the load will also be sinusoidal **iff** the load is linear.

Non-linear loads

- Diodes
- Superconductors
- Varistors (voltage-dependent resistors)
- Non-linear inductors

Generation of Electricity

Many forms of energy can be converted to electricity.

Sources of energy

- Pressure head of water
- Chemical energy of fuels
- Nuclear energy of radioactive substances
- Wind energy
- Solar energy

Hydro power

Potential energy of water is converted to mechanical energy using water turbines. The water turbines are connected to alternators which generate the alternating electricity.

Penstock

Large pipes laid on a slope. Carries water from a reservoir to a turbine. Used to increase the velocity of the water.

Surge tank

Aka. surge chamber. A cylindrical tank. Connected with penstock. Placed near the powerhouse. Used to control the pressure of water in penstock.

Main Inlet Valve (MIV)

A valve installed in between the penstock and the [turbine](#). Used to stop water in the middle for maintenance purposes.

Fuel energy

Examples for fuels:

- Coal (solid)
- Oil, Diesel (liquid)
- Natural gas (gas)

Heat energy of fuels is converted to mechanical energy (using steam turbines or internal combustion engines). Alternators are driven by the mechanical energy.

Nuclear energy

Heat produced during nuclear fission is used to drive the prime mover. Steam turbines are used here.

Wind energy

Wind turbines are the prime movers.

Common Terms

Branch

A branch represents a single element, such as a resistor or a battery.

Node

A node is the point connecting more than 1 branches. Denoted by a dot.

Note

All points in a circuit that are connected directly by ideal conductors can be considered to be a single node.

Two terminal element

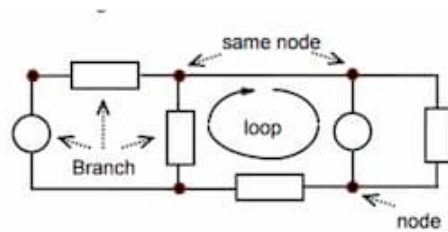
An element connected to two nodes. Branches are two terminal elements.

Loop

A loop is a closed path through a circuit in which no node is encountered more than once except the start/finish node.

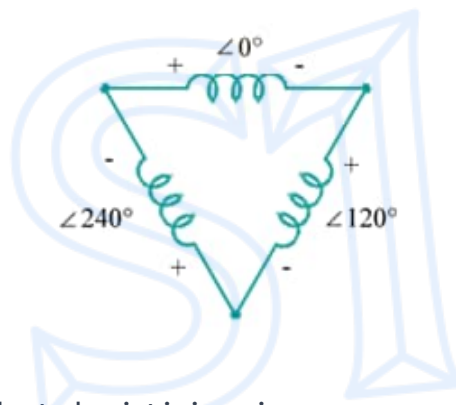
Mesh

A mesh is a loop without having other loops inside it. Subset of loops.



Connection types

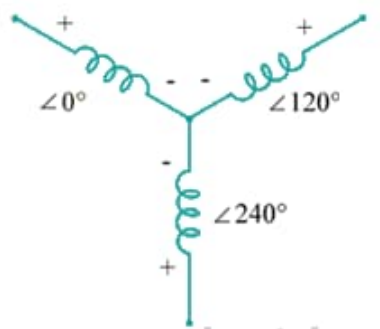
Delta connection



- Doesn't have a neutral wire. Neutral point is imaginary.
- Delivers more power

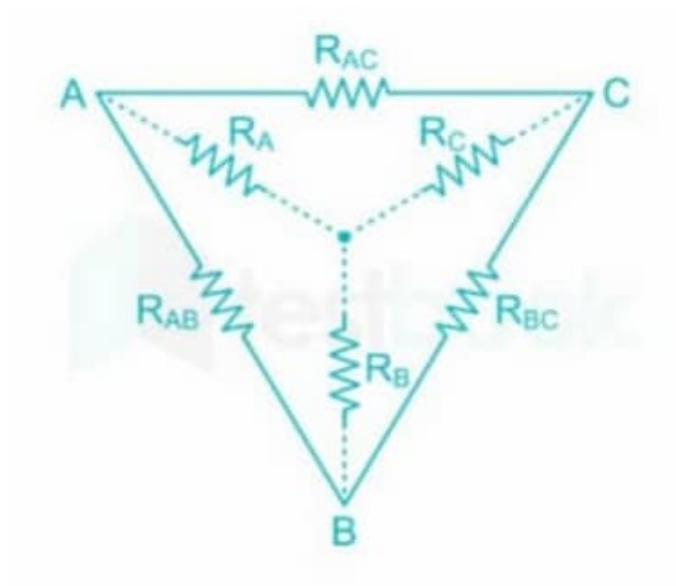
$$V_P = V_L \quad \wedge \quad I_L = \sqrt{3}I_P$$

Star connection



$$I_P = I_L \quad \wedge \quad V_L = \sqrt{3}V_P$$

Delta-Star conversion



Delta to Star

$$R_A = \frac{R_{AB} \times R_{AC}}{R_{AB} + R_{BC} + R_{AC}}$$

R_B, R_C can be found similarly.

Star to Delta

$$R_{AB} = \frac{(R_A \times R_B) + (R_B \times R_C) + (R_C \times R_A)}{R_C}$$

R_{BC}, R_{AC} can be found similarly.

Circuit elements

Two types of circuit elements.

- Active
- Passive

Active

Capable of generating electrical energy.

- Voltage sources
- Current sources

These can interchangeably be used.

Voltage sources

- Batteries - electrochemical
- Solar cells - photo voltaic
- Generators - electromagnetic

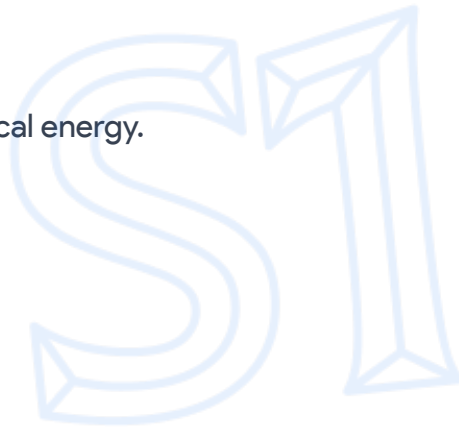
Ideal voltage source

Constant voltage for any required currents. Does not exist.

Passive

Either consumes or stores electrical energy.

- Resistors
- Inductors
- Capacitors
- Any other elements



Resistors

Resistance, in terms of physical dimensions:

$$R = \frac{\rho l}{A}$$

Here:

- l : length
- A : cross-sectional area
- ρ : [resistivity](#)

If a voltage V is applied across a conductor, then a given current I will flow through the conductor $V \propto I$. The proportionality constant is called resistance R .

$$R = \frac{V}{I}$$

Capacitors

Made of two conductive plates separated by an insulating ([dielectric](#)) layer.

Capacitance (C), in terms of physical dimensions:

$$C = \frac{\epsilon A}{d}$$

Here:

- ϵ : permittivity of the material in-between
- d : distance between the plates
- A : area of a plate

In an ideal capacitor, the charge imbalance Q is proportional to the voltage V across the plates.

$$Q = CV$$

v and i

As C is constant, current i passing through the capacitor and the voltage v across the capacitor are related by:

$$i = C \frac{dv}{dt}$$

Energy stored

Suppose voltage across an initially uncharged capacitor rises from 0 to V during a time period of t .

$$e = \int_0^t p dt = \int_0^t vi dt = C \int_0^v v dv$$

$$E = \frac{1}{2} CV^2$$

Inductors

When there is a current in the inductor, a magnetic field is created. Any change in current causes the magnetic field to change, this in turn induces a voltage across the inductor that opposes the original change in current.

A length of wire turned into a coil works as an inductor.

Inductance (L)

For an ideal inductor:

$$v = L \frac{di}{dt}$$

Here the v is the voltage difference between the inductor, and i is the current through the inductor.

The polarity is such as to oppose the change in current.

Energy stored

Suppose voltage across an inductor rises from 0 to i during a time period of t seconds.

$$e = \int_0^t p dt = \int_0^t vi dt = L \int_0^i i di$$

$$E = \frac{1}{2} Li^2$$

Kirchhoff Laws

Kirchhoff Current Law

The algebraic sum of all the currents entering and leaving a node is zero. Based on principle of conservation of charge.

$$\sum_{\text{node}} I = 0 \implies \sum_{\text{in}} I = \sum_{\text{out}} I$$

Kirchhoff Voltage Law

The algebraic sum of voltages around a loop is zero. Based on principle of conservation of energy.

$$\sum_{\text{node}} V = 0$$

Voltage division

Series connection is used to divide voltage. Potentiometers are commonly used to create voltage divider circuits.

Current division

Parallel connection is used to divide current.

Waves

Waveform

Obtained by plotting instantaneous values of a time-varying quantity against time.

Periodic Waveform

A pattern repeats after T time. Periodic time is T and frequency f is $\frac{1}{T}$.

Alternating Waveform

A waveform that changes in magnitude and direction with time. Is also a periodic waveform.

Symmetric Waveform

A periodic waveform that is symmetric about the time axis.

Sinusoidal Waves

An alternating and symmetric waveform. Same as $\sin \theta$ vs θ (in rad). Also called sine waves or sinusoid.

$$y = A \sin(\omega t + \phi)$$

When ϕ is:

- > 0 - the wave is said to be **leading** by ϕ
- $= 0$ - the wave is the **reference**
- < 0 - the wave is said to be **lagging** by ϕ

Sinusoidal voltages can be easily generated using rotating machines.

Complex Waveforms

Periodic non-sinusoidal waveforms can be split into its fundamental and harmonics.

Fundamental Waveform

$$f_0 = f_{\text{complex}}$$

Harmonics

Sine waves with higher frequencies which is a multiple of f_0 .

$$f_{\text{harmonic}} = n \cdot f_0 ; n \in \mathbb{Z}$$

Harmonics are grouped into:

- **odd harmonic** when n is odd.
- **even harmonic** when n is even.

AC Theory

ⓘ Note

Only sinusoidal AC supply are considered in s1.

Say v is alternating as in $v = V_m \sin(\omega t + \phi)$.

Peak value

Maximum instantaneous value. V_m in the example.

Peak-to-peak value

Maximum variation between maximum positive and negative instantaneous values. For a sinusoidal waveform, this is twice the peak value. $2V_m$ in the example.

Mean value

$$v_{\text{mean}} = \frac{1}{T} \int_{T_0}^{T_0+T} v(t) dt$$

Here:

- T_0 is the starting time of a cycle
- T is the periodic time

For any symmetric waveform, mean value is 0.

Average value

Mean value of the rectified version of a waveform.

For symmetric waveforms, half-cycle mean value is taken as the average value.

$$v_{\text{average}} = \frac{2}{T} \int_{T_0}^{T_0+\frac{T}{2}} v(t) dt$$

For sinusoidal waveforms, from the example:

$$\begin{aligned} v_{\text{average}} &= \frac{2}{T} \int_{T_0}^{T_0+\frac{T}{2}} V_m \sin(\omega t + \phi) dt \\ &= \frac{2}{\pi} V_m = 0.637V_m \end{aligned}$$

rms value

Aka. effective value. rms value is always used to express the magnitude of a time varying quantity.

$$v_{\text{rms}} = \sqrt{\frac{1}{T} \int_{T_0}^{T_0+T} v(t)^2 dt}$$

For sinusoidal waveforms:

$$v_{\text{rms}} = V_m \sqrt{\frac{1}{T} \int_{T_0}^{T_0+T} \sin^2(\omega t + \phi) dt} = \frac{V_m}{\sqrt{2}}$$

ⓘ Note

i_{rms} is the equivalent current that dissipates same amount of power across a resistor R in time T as $i(t)$. Similar for voltage.

Instantaneous power

$$P = vi = i^2 R$$

Form factor

$$\text{Form factor} = \frac{\text{rms value}}{\text{average value}} = \frac{V_m}{\frac{2}{\pi V_m}} = 1.111$$

Peak factor

$$\text{Peak factor} = \frac{\text{peak value}}{\text{rms value}} = V_m \times \frac{\sqrt{2}}{V_m} = 1.412$$

Phasor Representation

Phasor (phase vector) is a vector representing a sinusoidal function.

- Magnitude of the phasor: rms value of the wave
- Angle of the phasor: The angular position ϕ , with respect to a reference direction

Can also be represented by a complex number. But j is used as the imaginary unit because i is used for time-varying current.

Representation

- Polar form: $A = |A|\angle\phi$
- Cartesian or rectangular form: $A = A_x + jA_y$

Here:

- $|A| = A_{\text{rms}} = \sqrt{A_x^2 + A_y^2}$
- $A_x = |A| \cos \phi$
- $A_y = |A| \sin \phi$
- $j = \sqrt{-1}$
- $\tan \phi = \frac{A_y}{A_x}$

Impedance & Admittance

Impedance (Z)

$$Z = \frac{V}{I} = R + jX$$

Here:

- R : Resistance
- X : Reactance

(i) Note

When mentioning the reactance of an element, the j should not be included.

Admittance (Y)

Inverse of impedance.

$$Y = \frac{1}{Z} = \frac{I}{V} = G + jB$$

Here:

- G : Conductance
- B : Susceptance

From the definitions:

$$G = \frac{R}{R^2 + X^2} \quad \wedge \quad B = -\frac{X}{R^2 + X^2}$$

For simple circuit elements

Resistor

Let $i = I_m \sin(\omega t + \phi_0)$ is applied across a resistor with resistance R . From Ohm's law:

$$v = RI_m \sin(\omega t + \phi_0) \implies Z_R = R$$

No changes in frequency, phase angle. v is in phase with i . R doesn't have reactance.

Inductor

Let $i = I_m \sin(\omega t + \phi_0)$ is applied across an inductor with inductance L .

$$v = L\omega I_m \sin\left(\omega t + \left(\phi_0 + \frac{\pi}{2}\right)\right) \implies Z_L = j\omega L$$

Reactance of the inductor is $X_L = L\omega$.

ⓘ Note

v leads i by $\frac{\pi}{2}$. No changes in frequency.

Capacitor

Let $i = I_m \sin(\omega t + \phi_0)$ is applied across a capacitor with capacitance c .

$$v = \frac{I_m}{c\omega} \sin(\omega t + (\phi_0 - \frac{\pi}{2})) \implies Z_C = -j \frac{1}{c\omega}$$

Reactance of the capacitor (capacitive reactance) is $X_c = -\frac{1}{c\omega}$.

Note

v lags i by $\frac{\pi}{2}$. No changes in frequency.

Note

If v :

- lags i - circuit is capacitive
- leads i - circuit is inductive

For complex circuit elements

For a series circuit

Resultant impedance is the sum of each component's impedance.

For a parallel circuit

Resultant admittance is the sum of each component's admittance.

Power and Power factor

- In a purely resistive AC circuit, the energy delivered by the source will be dissipated as heat by the resistor.
- In a purely capacitive or purely inductive circuit, all of the energy will be stored during a half cycle, and then returned to the source during the other – there will be no net conversion to heat.
- When there is both a resistive component and a reactive component, some energy will be stored, and some will be converted to heat during each cycle.

Power equations

Purely resistive circuit

Suppose a circuit with load R resistance is supplied a voltage of $v(t) = V_m \cos \omega t$.

Instantaneous power dissipated by the load is given by:

$$p(t) = \frac{V_m^2}{R} \cos^2(\omega t)$$

Always: $p(t) > 0$.

$$\text{Average power} = \frac{1}{2} \times \text{Peak power} = \frac{V_m^2}{2R}$$

Purely inductive circuit

Suppose a circuit with inductor L is supplied a voltage of $v(t) = V_m \cos \omega t$.

Instantaneous power dissipated by the load is given by:

$$p(t) = \frac{V_m^2}{2\omega L} \sin(2\omega t)$$

Purely capacitive circuit

Suppose a circuit with inductor L is supplied a voltage of $v(t) = V_m \cos \omega t$.

Instantaneous power dissipated by the load is given by:

$$p(t) = -\frac{V_m^2 \omega C}{2} \sin(2\omega t)$$

The capacitive reactive power is given by:

$$Q = V^2 \omega C$$

General load

Consider a general load with both resistive and reactive components. Depending on how inductive or capacitive the reactive component, the phase shift between voltage and current phasor lies between 90° and -90° .

Suppose the circuit is supplied a voltage of $v(t) = V_m \cos(\omega t)$. And the current phasor shifts in θ phase angle.

$$i(t) = I_m \cos(\omega t - \theta)$$

This ends up with:

$$p(t) = \frac{1}{2} V_m I_m \left[\cos \theta + \cos(2\omega t - \theta) \right]$$

Average over 1 cycle

$$P_{\text{avg}} = \frac{1}{T} \int_{t_0}^{t_0+T} p(t) dt = V_{\text{rms}} I_{\text{rms}} \cos \theta$$

Types of power

Active power

Aka. true power, resistive power. In all electrical and electronic systems, it is the true power (the resistive power) that does the work.

$$P = V_{\text{rms}} I_{\text{rms}} \cos \theta$$

ⓘ Note

In a question, if “power” is asked to be calculated, that means “active power”.

Reactive Power

Power delivered to/from a pure energy storage element (inductors and capacitors) is known as reactive power.

- Average power consumed by a pure energy storage element is 0.
- Current associated with it is **not** 0. Transmission lines, transformers, fuses, etc. must all be designed to be capable of withstanding this current.
- Loads with energy storage elements will draw large currents and require heavy duty wiring even though little average power is consumed.
- Shuttles back and forth between the source and the load.

$$Q_{\text{reactive}} = V_{\text{rms}} I_{\text{rms}} \sin \theta$$

Apparent power

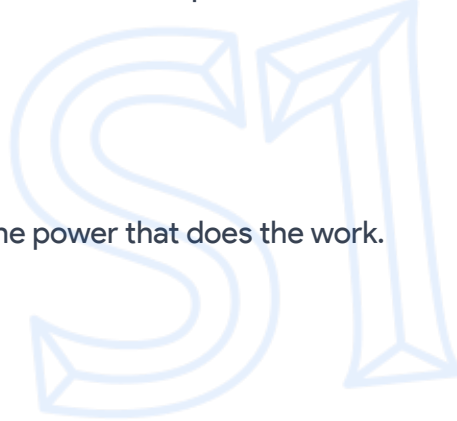
Combination of active and reactive power.

$$S = V_{\text{rms}} I_{\text{rms}} = \sqrt{P^2 + Q^2}$$

The apparent power is essentially the effective power that the source “sees”.

The Beer Analogy

- Beer - Active power
Liquid beer is useful power. The power that does the work.
- Foam - Reactive power
Wasted or lost power.
- Mug - Apparent power
Demand power, that is being delivered by the utility.



Power factor

If θ is the phase angle difference between v and i , $\cos(\theta)$ is called the power factor.

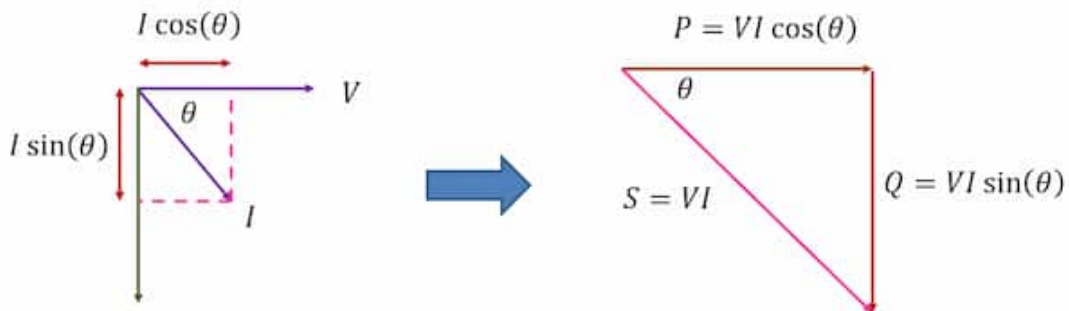
Power factor appears in the equation of P_{avg} .

$$\cos \theta = \frac{\text{Active power}}{\text{Apparent power}} = \frac{\text{Resistance}}{\text{Impedance}}$$

Power factor is:

- leading when I leads V
- lagging when I lags V

Power triangle



- Take V phasor as the reference.
- Draw V and I phasors.

Power systems

An electric power system consists of 3 principle sections.

- Power stations: electricity is generated
- Transmission: voltage is stepped up to high voltage
- Distribution: voltage is stepped down to medium voltage for distribution over a relatively small region

Variable load

Load on a power station changes with to uncertain demands of consumers. This is called the **variable load**.

Load vs time curve is called the **load curve**. Area under this curve is the **total energy requirement**.

Power grid

Nation-wide, massive, geographically distributed system for electrical power supply network.

Sri Lankan Scenario

There are 2 major electric utilities in Sri Lanka: CEB and LECO.

Voltage Levels

- Generation voltage **13.8 kV**
- High voltage **132 kV** or **220 kV**
- Medium voltage **11 kV** or **33 kV**
- Nominal voltage **230 V**
- Nominal line-to-line **400 V**

Power sources

Hydro-power is the most used source of energy for electricity.

Note

Public Utilities Commission of Sri Lanka (PUCSL) is the economic, technical and safety regulator of the electricity industry in Sri Lanka.

Why high voltage transmission?

Reduction of power losses

$$P_{\text{loss}} = I^2 R$$

Power losses in transmission lines are called " $I^2 R$ " losses. To reduce the power loss, current have to be reduced.

Power generation in the generator is constant. So voltage is increased to reduce the current.

Reduce voltage drop

Voltage drop in the transmission lines is proportional to the current flowing through it.

Reduction of power transmission cost

To carry higher currents, the transmission lines must have higher cross sectional area. Reduced current means smaller transmission lines are enough. Which leads to reduced cost.

Lab Apparatus

Multimeter

Measures voltage, current, frequency.

There are 3 variants:

- Moving coil meter
- Moving iron meter
- Moving rectifier meter

Oscilloscope

Aka O'scope. Graphically displays electrical signals and how they vary with time. Used when designing, manufacturing, and repairing electrical equipment. Comes in either 1 or 2 or 4 channels. Each channel is used to connect a probe using BNC connector. Vertical section controls the voltage or current scale for each channel. Horizontal section controls the time scale.

Passive Voltage Probes

Measures voltage difference between ground clip and hook clip. Hook is covered with a retractable cover. The BNC connector includes a capacitor compensation trimmer.

Each channel inside the oscilloscope includes a $1\text{ M}\Omega$ resistor and 16 pF capacitor. Works for low frequencies. Picks up noises for higher frequencies. To mitigate this, a coax cable is included in the probe.

Most probes include a switch between $1x$ and $10x$, which refers to how much the amplitude of the signal is attenuated (reduced). The $10x$ setting has an additional $9\text{ M}\Omega$ resistor and a compensation capacitor.

Calibration

- Set 10x on the probe
- Connect it to the oscilloscope
- Make sure only the appropriate channel is turned on
- Make sure DC option is selected in Coupling setting
To see all parts of the signal
- Select 10x option in Probe setting
- Make sure Type setting is selected to Edge
- Make sure Source setting is selected to the appropriate channel
- Attach ground clip to the ground tab
- Hook the probe around the signal tab
- Adjust the vertical and horizontal scale knobs
To clearly see the waveform. Turning the knobs clockwise decreases the time or voltage scale (like zooming in).
- Adjust the smaller knob of channel 1 To center the waveform vertically
- Use the knob in the Trigger section To raise the level somewhere between the min and max of the waveform
- Adjust the capacitor compensation trimmer
Until the square wave has straight edges

While visualizing 2 voltages of a circuit, the ground clip is connected to the reference point. Only 1 ground clip of the multiple probes must be connected to prevent short circuits inside the oscilloscope.

Current Probe

Operation is similar to current transformer. Steps down the current, converts to a voltage signal, and renders it in the oscilloscope. Has a sensor head, which can be opened by the opening lever. Powered through power supply cable.

Calibration

- Connect the BNC connector to a channel
- Connect power supply cable into a power supply port in the rear side of the oscilloscope
- Connect the sensor head to the circuit
- Turn on the channel
-

Variac

A variable AC power supply. Input is connected to **230 V** AC power supply.

Rheostat

A variable resistor. Used to control the current in a circuit.

3-Phased System

Why 3-phase?

- The current can be distributed into 3 wires instead of just 1.
There is a maximum limit of how much current a wire can carry.
- Economical as less amount of wires.
3-phase system requires **4** wires (**3** if balanced) while single phase system requires **6**.

The phases are denoted by **R, Y, B** in that order.

Balanced 3-phase

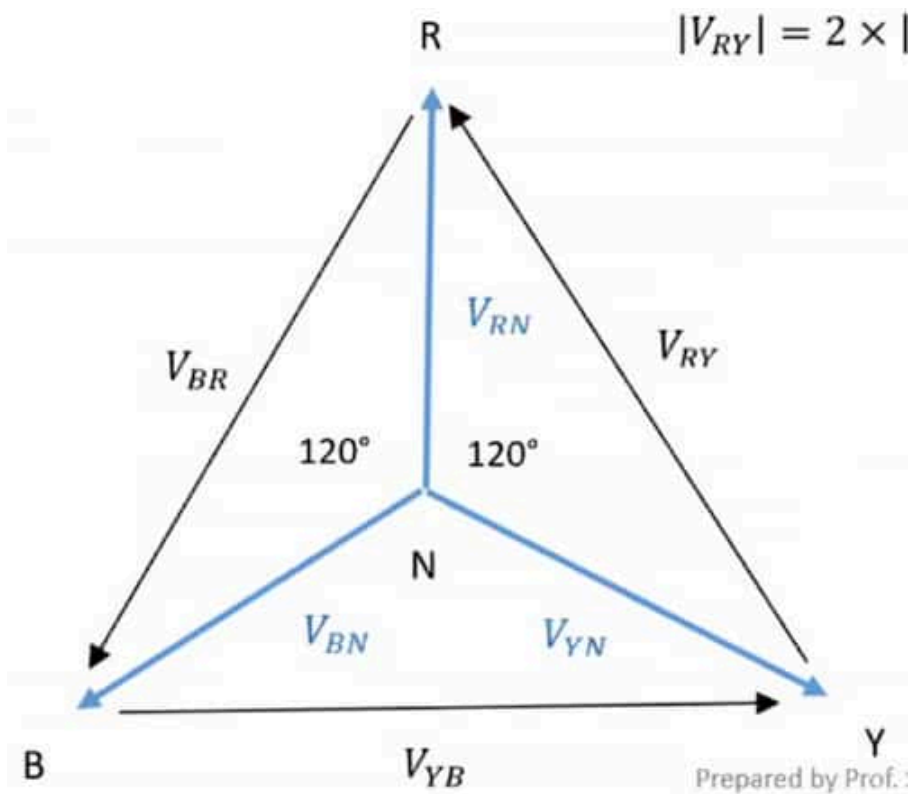
A 3-phase system is said to be balanced **iff**:

- Supply is balanced
- Loads are the same in each phase

Power source

A 3-phase power source which produces 3 phase voltages of equal rms value, but with **120°** phase difference.

Phasor diagram



Phase voltage

Voltage between a phase wire and the neutral wire.

V_{RN} , V_{YN} , V_{BN} are the phase voltages.

Line-to-line voltage

Voltage between any 2 phase wires. Line-to-line voltages also have a 120° phase difference.

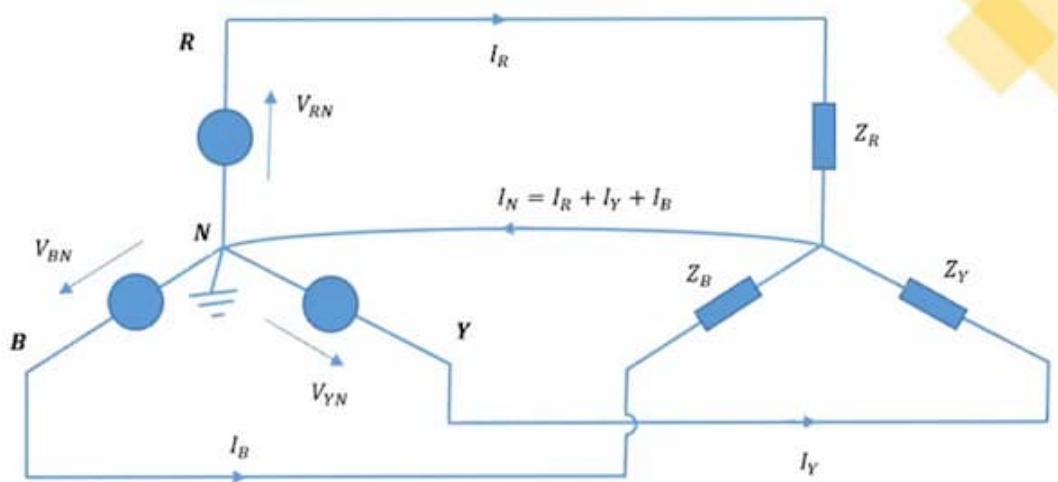
V_{RY} , V_{YB} , V_{BR} are the line-to-line voltages or line voltages.

$$|V_{BR}| = 2 \times |V_{BN}| \cos(30^\circ) = \sqrt{3} |V_{BN}|$$

ⓘ Note

In a 3-phase system, line-to-line voltage is mentioned.

Analysis



$$I_N = E \left[\frac{1 \angle 0^\circ}{z_R} + \frac{1 \angle -120^\circ}{z_Y} + \frac{1 \angle 120^\circ}{z_B} \right]$$

When the loads are balanced: $z_R = z_Y = z_B = z$, $I_N = 0$

In this case, neutral wire is optional and can be eliminated. $I_N = 0$ have to be maintained so that the voltage is equal to ground voltage in neutral wire. This makes sure there are no power losses in neutral wire.

Real-life Usage

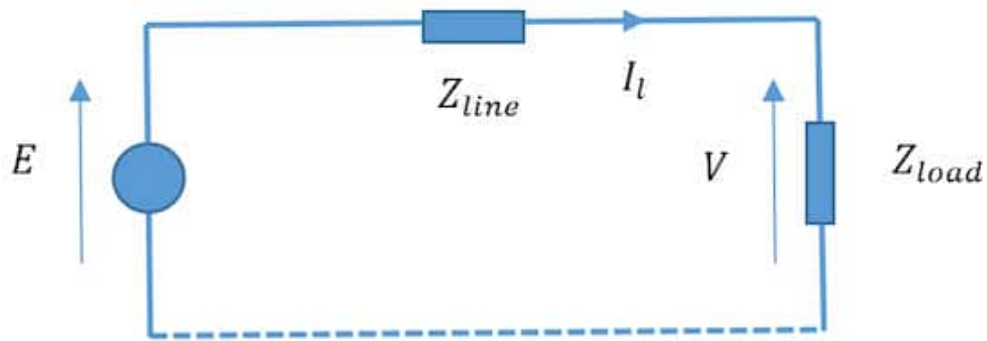
Most domestic loads are single-phase. In case of 3-phase domestic wiring, the single-phase loads are distributed among the 3 phases at the main distribution board.

Devices that have a 3-phase power input, doesn't require a neutral line.

Per-phase Equivalent Circuit

Power, voltage, current, power factor are same for all 3 phases.

When a 3-phase system is balanced, it is sufficient to consider only a single phase. The diagram showing the single-phase equivalent of the power system using standard symbols.



Here:

- E - voltage across the source
- V - voltage across the load

$$\text{Per-phase power} = |V_p||I_l| \cos \theta = \frac{1}{3} \times \text{3-phase power} \quad \wedge \quad |V_l| = \sqrt{3}|V_p|$$

$$\Rightarrow \text{3-phase power} = \sqrt{3}|V_l||I_l| \cos \theta$$

Here:

- V_p - phase voltage
- V_l - line voltage
- I_l - line current
- $\cos \theta$ - power factor
- The power can either be source power, load power, or transmission power losses.

Unbalanced 3-phase system

A 3-phase system becomes unbalanced, when load distribution is not equal among the phases. $I_N \neq 0$. Highly undesirable. Neutral wire is the return path for the line currents and is compulsory.

Large currents in the neutral wire could cause:

- If neutral wire have significant impedance, different points of the neutral wire will have different voltage
- Series voltage unbalances can happen if the neutral wire is broken

Each phase will be different. Complete system has to be considered when analyzing the circuit.

Note

Analysis of unbalanced 3-phase systems is not required in s1.

