

# Dynamics

## Introduction

A branch of mechanics, which deals with motion of bodies.

2 parts:

- **Kinematics:** the study of geometric aspects of motion (not referencing the forces)
- **Kinetics:** the analysis of the forces that cause the motion

## Kinematics of a particle

A particle has a mass and negligible size.

### Note

When bodies of finite size is of interest, the body might be considered as particles **provided** motion of the body is characterized by motion of its center of mass and any rotation of the body is neglected.

## Rectilinear motion

When the motion of a particle is along a straight line.

Suppose  $x$  is the distance to the particle from a fixed point on its motion path.

- $\dot{x}$  is its instantaneous velocity.
- $\ddot{x}$  is its instantaneous acceleration.

## Curvilinear motion

When the motion of a particle is along a curve.

Suppose  $\vec{r}$  is the position vector of the particle.

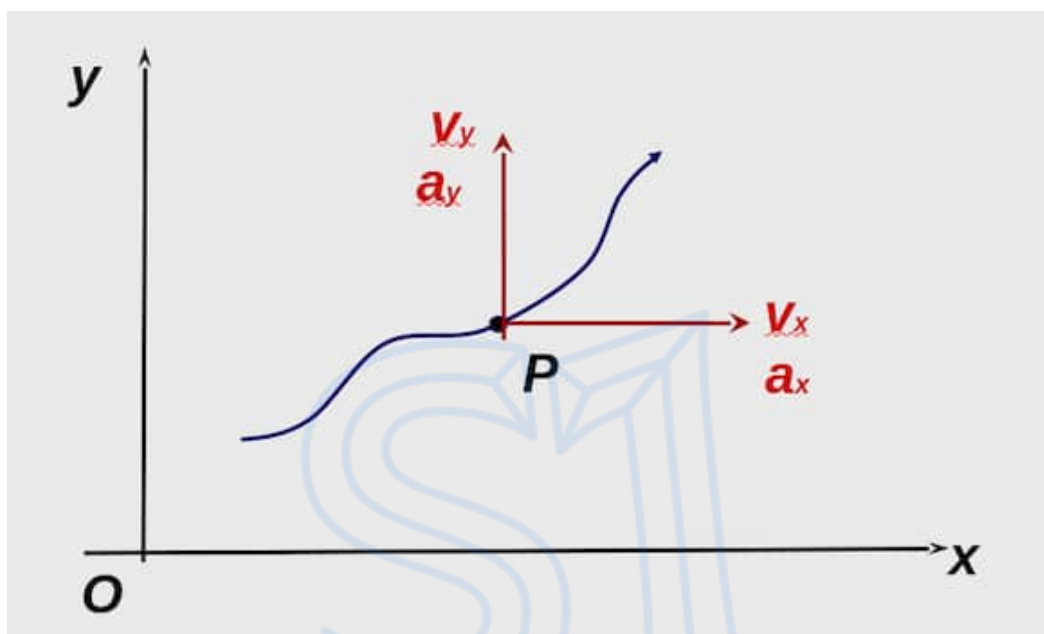
- Instantaneous velocity  $v = \frac{dr}{dt}$
- Instantaneous speed  $|v| = \frac{ds}{dt}$
- Instantaneous acceleration  $a = \frac{dv}{dt}$

ⓘ Note

Right hand rule is used here to denote the direction of any rotary motions.

## 2D motion of a particle

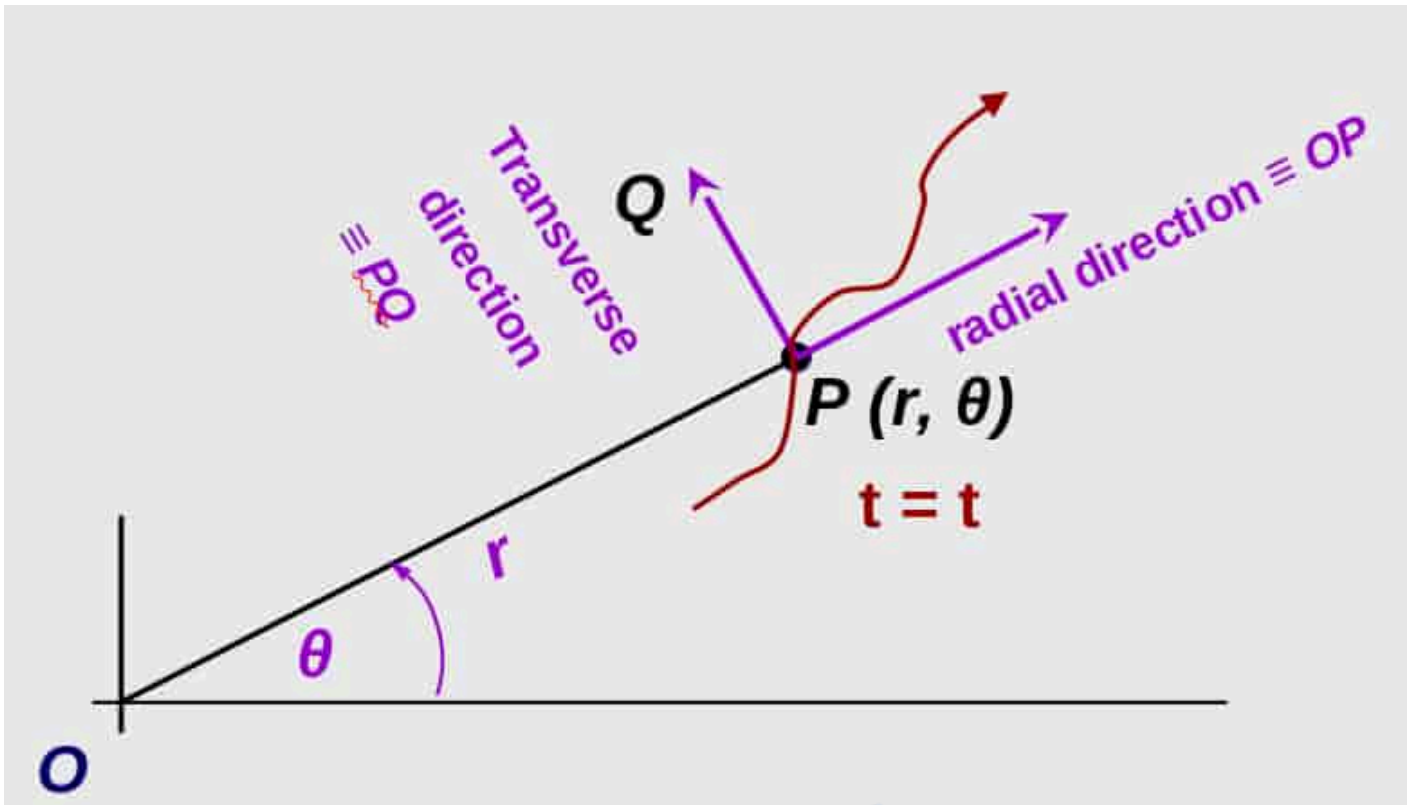
### Rectangular form



$$v_y = \frac{dy}{dt} = \dot{y} \quad \wedge \quad v_x = \frac{dx}{dt} = \dot{x}$$

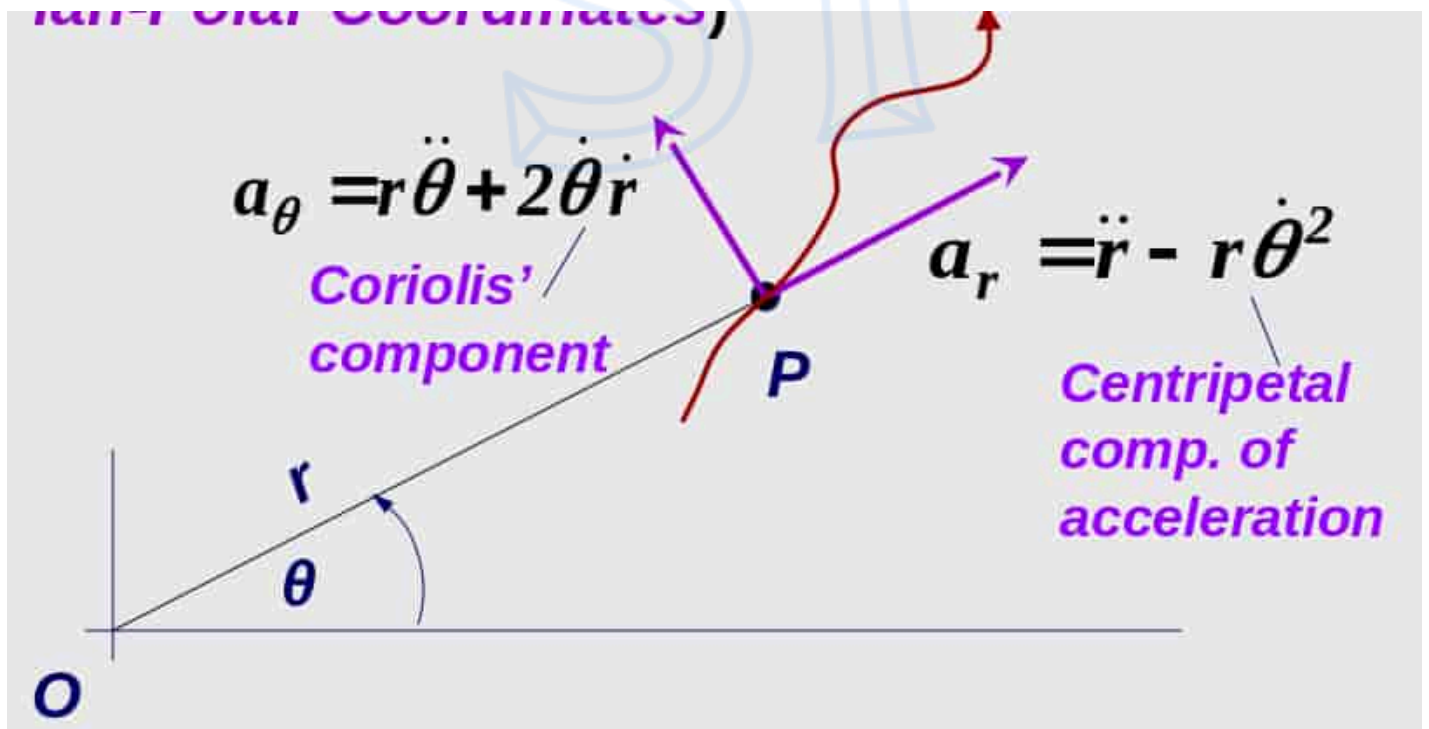
$$a_y = \frac{d^2y}{dt^2} = \ddot{y} \quad \wedge \quad a_x = \frac{d^2x}{dt^2} = \ddot{x}$$

Polar form



Velocity have a transverse and radial components.

- Transverse component  $v_\theta = \dot{\theta} \times r$
- Radial component  $v_r = \dot{r}$



Acceleration also have a transverse and radial components.

- Transverse component
  - $a_\theta = r\ddot{\theta} + 2\dot{\theta}\dot{r}$
  - In vector equation:  $\underline{a}_\theta = \underline{\ddot{\theta}} \times \underline{r} + 2(\underline{\dot{\theta}} \times \underline{\dot{r}})$
- Radial component
  - $a_r = \ddot{r} - r\dot{\theta}^2$
  - $\underline{a}_r = \underline{\ddot{r}} + \underline{\dot{\theta}} \times (\underline{\dot{\theta}} \times \underline{r})$

In the acceleration:

- Coriolis' component of acceleration:  $2\dot{\theta}\dot{r}$
- Centripetal component of acceleration:  $-r\dot{\theta}^2 = \underline{\dot{\theta}} \times (\underline{\dot{\theta}} \times \underline{r})$

### Effects of Coriolis' component

- Objects deflect to the right in the northern hemisphere
- Objects deflect to the left in the southern hemisphere
- Maximum deflections occur at the poles. No deflection at the equator.

### Unit vectors

Unit vectors in transverse and radial directions are denoted by  $e_\theta$  and  $e_r$  respectively.

$$\dot{e}_r = \dot{\theta}e_\theta \quad \wedge \quad \dot{e}_\theta = -\dot{\theta}e_r$$

### Velocity

$$v = \frac{d}{dt}(re_r) = \dot{r}e_r + r\dot{e}_r = \dot{r}e_r + r\dot{\theta}e_\theta$$

### Acceleration

$$a = \frac{d}{dt}(r\dot{\theta}e_\theta) = (\ddot{r} - r\dot{\theta}^2)e_r + (r\ddot{\theta} + 2\dot{\theta}\dot{r})e_\theta$$

## 2D kinematics of a rigid body

### Rigid body

A solid body that doesn't deform.

## Degrees of freedom

In the motion of a rigid body in 2D kinematics, there are 3 degrees of freedom.

- Movement along  $x$  direction
- Movement along  $y$  direction
- Rotation about  $z$  direction

In 3D, there are 6 degrees of freedom: movement and rotation along each direction.

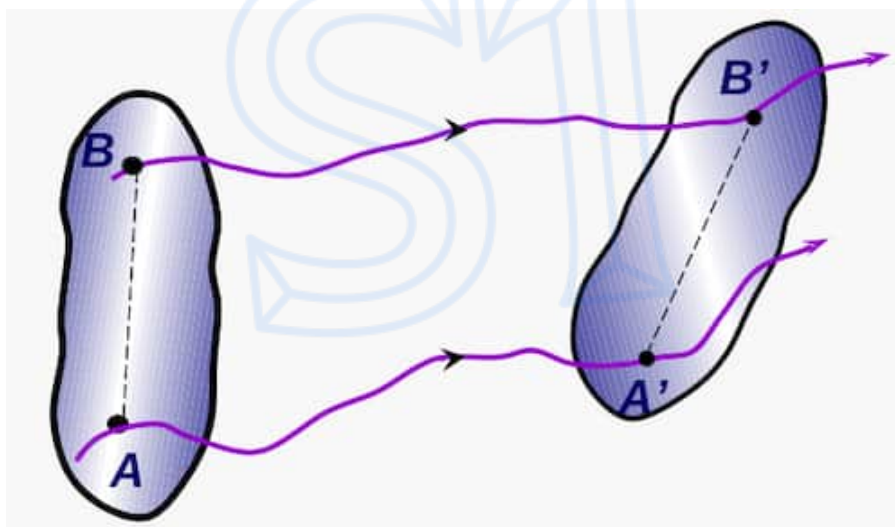
## Translation

Movement that changes the position of an object. Translation can be done through a rectilinear or curvilinear path. Axes of the body always stays parallel.

## Rotation

Circular movement of an object about a fixed axis that is perpendicular to the plane.

## General 2D motion



Mixture of translation and rotation.

$$v_B = v_A + v_{B/A} = v_A + \underline{\dot{\theta}} \times r_{B/A}$$

$$a_B = a_A + a_{B/A} = a_A + \underline{\ddot{\theta}} \times r_{B/A} + \underline{\dot{\theta}} \times (\underline{\dot{\theta}} \times r_{B/A})$$

Here:

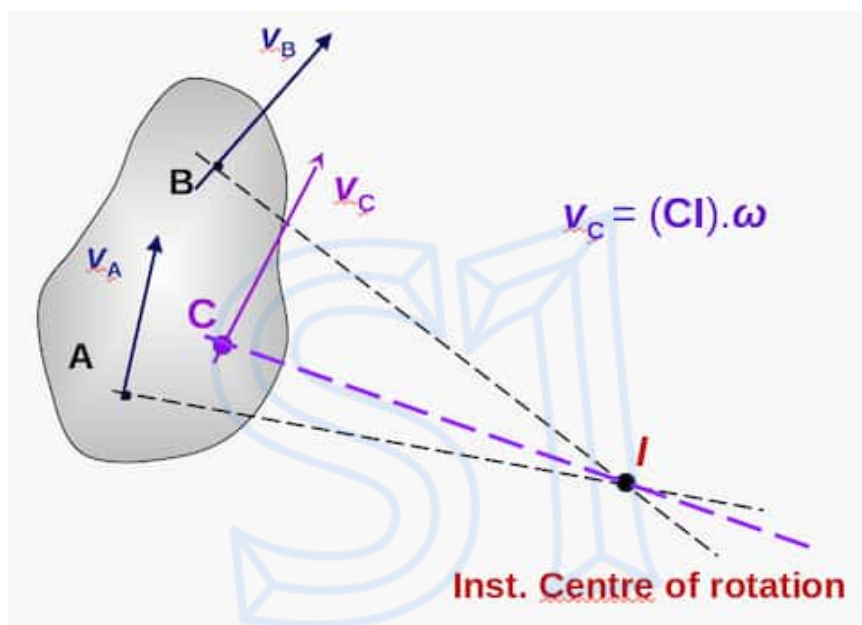
- $\dot{\theta}$  - Angular velocity of **B** relative to **A**
- $v_{B/A}$  - Velocity of **B** relative to **A**
- $a_{B/A}$  - Acceleration of **B** relative to **A**
- $r_{B/A}$  - Position vector of **B** relative to **A**. It's constant.

In general motion, each particle of the body has a different velocity at every instance.

### Instantaneous centre of rotation

The point that has 0 velocity at a particular instant of time. This point might be changing throughout the motion. Denoted by  $I$ .

It can be imagined that the object is momentarily having a pure rotation about this centre  $I$ .



$I$  can be found by drawing a line perpendicular at the velocity vectors at 2 different points and finding their intersection point.

### Centrode

The locus of instantaneous centres during the motion.

## Mechanisms

### Mechanism

An assembly of rigid bodies or links designed to obtain a desired motion from an available motion while transmitting appropriate forces and moments. Motion of the links have definite relative motion with other links.

## Simple mechanisms

- Lever
- Pulley
- Gear trains
- Belt and chain drive
- Four bar linkage

## Other complex mechanisms

- Lock stitch mechanism (used in sewing machine)
- Geneva mechanism  
Constant rotational motion to intermittent rotational motion. mostly used in watches.
- Scotch yoke mechanism  
Constant rotational motion to linear motion (vice versa.). Mainly used as valve actuators in high pressure gas pipelines.
- Slider crank mechanism  
Used in internal combustion engines

## 2D link mechanisms

### Method of instantaneous centre of rotation

- Find the instantaneous centre of the rotation from known velocities at known points
- Use the instantaneous centre to find velocities at other points

### Kinematic chain

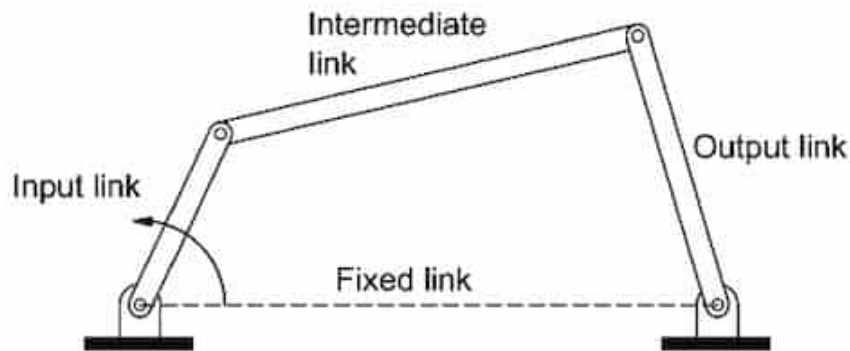
An arbitrary collection of links (forming a closed link) that is capable of relative motion and that can be made into a rigid structure by an additional single link.

## Four-bar Mechanism

Four bar-shaped members connected to each other in one plane.

Usually:

- 1 fixed link + 3 moving links
- 4 pin joints
- 2 moving pivots + 2 fixed pivots
- 4 turning pairs



- **input link** - usually denoted in the left.
- **output link** - usually denoted in the right.
- **coupler** - intermediate link
- **frame** - fixed link

## Grashof's law

A four bar mechanism has at least one revolving link **if**  $l_0 + l_3 \leq l_1 + l_2$ .

Here:  $l_0, l_1, l_2, l_3$  are the length of four bars from shortest to longest.

## Modes of motions

Mechanism	Shortest link	Criteria
Crank rocker	Input link	$s + l < p + q$
Double crank	Fixed link	$s + l < p + q$
Double rocker	Coupler link	$s + l < p + q$
Change point	Any	$s + l = p + q$
Triple rocker	Any	$s + l > p + q$

**crank** means a link that makes a full revolution. **rocker** means a link that doesn't make a full revolution.



## Crank rocker mechanism

Shortest link rotates a full revolution. Output link oscillates.

## Double crank mechanism

Shortest link is fixed. Both input and output links rotate a full revolution.

## Double rocker mechanism

Shortest link makes full revolution. Input and output links make a full revolution.

## Special cases

$$l_0 + l_3 = l_1 + l_2.$$

Mechanism	Orientation
Parallelogram linkage or anti-parallelogram linkage	Equal links are opposite to each other
Deltoid linkage	Equal links are adjacent to each other

## Parallelogram linkage

Double crank mechanism. Opposite links are equal and parallel. Angular velocity of input crank & output crank is same. Orientation of the coupler doesn't change during the motion.

## Anti-parallelogram linkage

Double crank mechanism. Angular velocity of input crank is different to output crank.

## Deltoid linkage

- Longest link is fixed: crank rocker mechanism
- Shortest link is fixed: double crank mechanism

## Non-Grashof's condition

A four bar mechanism with the property **if**  $l_0 + l_3 > l_1 + l_2$ .

Here:  $l_0, l_1, l_2, l_3$  are the length of four bars from shortest to longest.

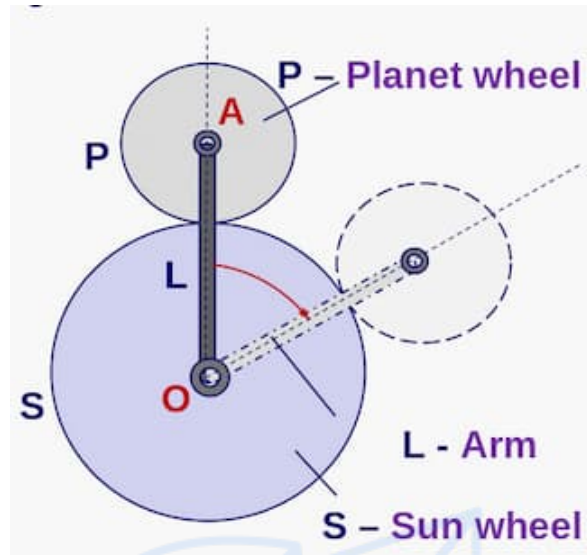
Three links are in oscillation.

# Epicyclic Gears

In below equations:

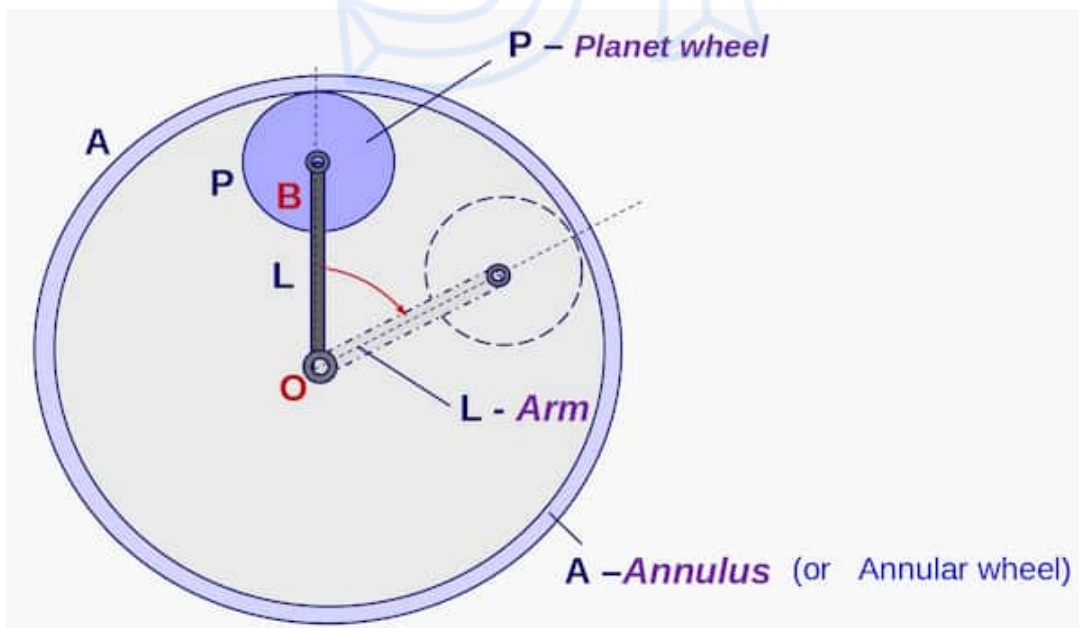
- $\omega_p$  - Absolute angular speed of planet wheel  $P$

## External



$$\omega_p = \left(1 + \frac{r_S}{r_P}\right)\omega_L - \left(\frac{r_S}{r_P}\right)\omega_S$$

## Internal



$$\omega_p = \left(1 - \frac{r_A}{r_P}\right)\omega_L + \left(\frac{r_A}{r_P}\right)\omega_A$$

# Mobility of Mechanisms

## Independent object

Has 3 degrees of freedom.

## Lower Pair

Aka. primary pair. A pair of kinematic elements which share a surface of contact.

When a rigid body is constrained by a lower pair, which allows only rotational or sliding movement. It has 1 degree of freedom, and the 2 degrees of freedom are lost.

Some examples:

- Turning pair
- Sliding pair
- Helical thread

## Higher Pair

A pair of kinematic elements which share only a line or a point of contact.

When a rigid body is constrained by a higher pair, it has 2 degrees of freedom: translating along the curved surface and turning about the instantaneous contact point.

Gear is an example.

When 2 independent objects are brought together to create a link, some degree of freedom will be lost.

“You lose some freedom when you become a couple.” — Our Dynamics Lecturer

## Grubler's Equation

Suppose  $N$  kinematic elements are brought together. 1 of them is fixed. The remaining elements have  $3(N - 1)$  degrees of freedom. Each lower pair loses 2 degrees of freedom. Each higher pair loses 1 degree of freedom. For a workable mechanism, resultant degrees of freedom must be 1.

$$F = 3(N - 1) - 2L - H = 1 \implies 3N - 2L + H = 4$$

Here:

- $F$  - degree of freedoms
- $N$  - number of kinematic elements
- $L$  - number of lower pairs
- $H$  - number of higher pairs

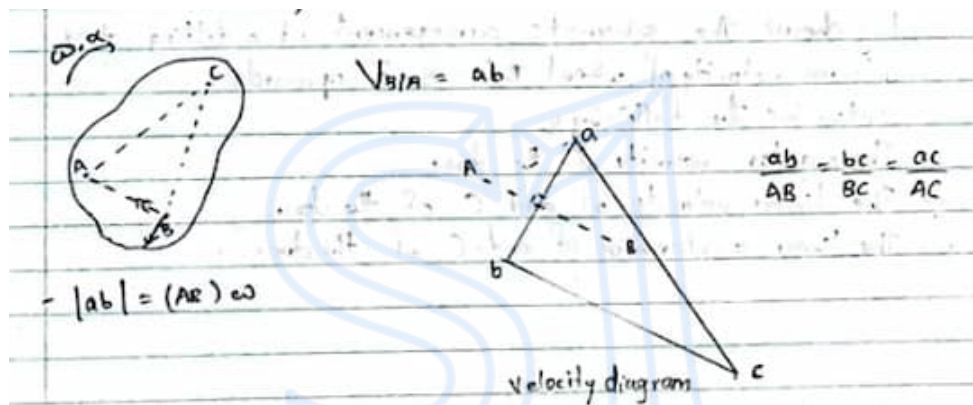
## Velocity & Acceleration Diagram

O is a fixed point.

### Velocity diagram

#### Notation

- $oa$  - Absolute velocity of point  $A$
- $ab$  - Velocity of point  $B$  relative to point  $A$



The above illustration is from Ruththiragayan, one of my friends.

### Acceleration diagram

#### Notation

- $o_1a_1$  - Absolute acceleration of point  $A$
- $a_1b_1$  - Velocity of point  $B$  relative to point  $A$

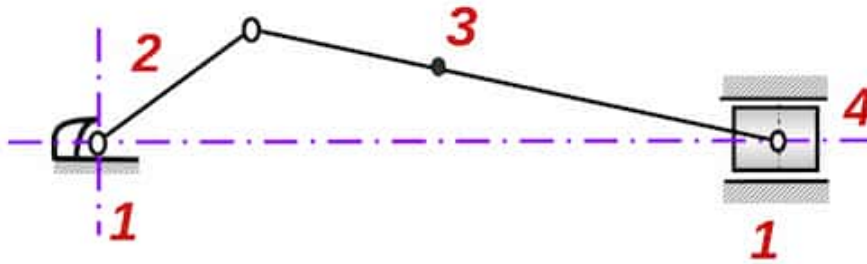
$$a_1b_1 = a_1x_1 + x_1b_1 = (AB)\omega_{AB}^2 + (AB)\alpha_{AB}$$

Here:

- $a_1x_1$  - the radial component of the relative acceleration between A and B
- $x_1b_1$  - the transverse component of the relative acceleration between A and B

# Inversions of a mechanism

The inversions are obtained by making different kinematic element stationary (one at a time) while keeping the same set of kinematic pairs.



For example, in slider crank mechanism:

- When link 2 is fixed: Whitworth quick-return mechanism
- When link 3 is fixed: The oscillating cylinder engine
- When link 4 is fixed: Hand pump

