Statics

Introduction

Centroid / Centre of area

The point where the area of a figure is assumed to be concentrated.

Located on the axes of symmetry.

First moment of area

Measure of spatial distribution of a shape in relation to an axis.

$$\text{About x-axis} = \int_A y \, \mathrm{d}A = A\bar{x}$$

$$\text{About y-axis} = \int_A x \, \mathrm{d}A = A\bar{y}$$

Here:

- $\bar{x}$ - Centroid's x coordinate
- $\bar{y}$ - Centroid's y coordinate
- A - Total area

About an axis of symmetry, first moment of area is 0.

Second moment of area

$$\text{About x-axis} = I_{xx} = I_x = \int_A y^2 \, \mathrm{d}A$$

$$\text{About y-axis} = I_{yy} = I_y = \int_A x^2 \, \mathrm{d}A$$

Always positive.

For common shapes

Shape	Description	I_{xx}
Rectangle or Parallelogram	Base b. Height h. About centroidal axis parallel to base.	$\frac{bh^3}{12}$
Triangle	Base b. Height h. About base.	$\frac{bh^3}{12}$
Triangle	Base b. Height h. About centroidal axis parallel to base.	$\frac{bh^3}{36}$
Circle	Diameter d. About centroidal axis.	$\frac{\pi d^4}{64}$

The product of moment of area about x,y axes

$$I_{xy} = \int_A xy \, \mathrm{d}A$$

The polar moment of area about z axis

$$I_{zz} = J_0 = \int_A r^2 \, \mathrm{d}A = I_{xx} + I_{yy}$$

Radius of gyration

$$\text{About x-axis} = r_x^2 = \frac{I_{xx}}{A}$$

$$\text{About y-axis} = r_y^2 = \frac{I_{yy}}{A}$$

$$\text{About z-axis} = r_z^2 = \frac{I_{zz}}{A}$$

Parallel Axis Theorem

$$I_x = I_{x_1} + A\bar{y}^2$$

$$I_y = I_{y_1} + A\bar{x}^2$$

$$I_{xy} = I_{x_1 y_1} + A\bar{x}\bar{y}$$

Here

- On LHS, the moments of area are about some x , y axes.
- On RHS, the moments of area are about centroidal axes x_1 , y_1 parallel to x, y.
- $\bar{x}$ is the distance between x and x_2 axes.
- $\bar{y}$ is the distance between y and y_1 axes.

> **Note**
>
> I_x is at a minimum when the axis is through the centroid. Same for I_y.

Perpendicular Axis Theorem

$$I_{zz} = I_{xx} + I_{yy} = I_{mm} + I_{nn}$$

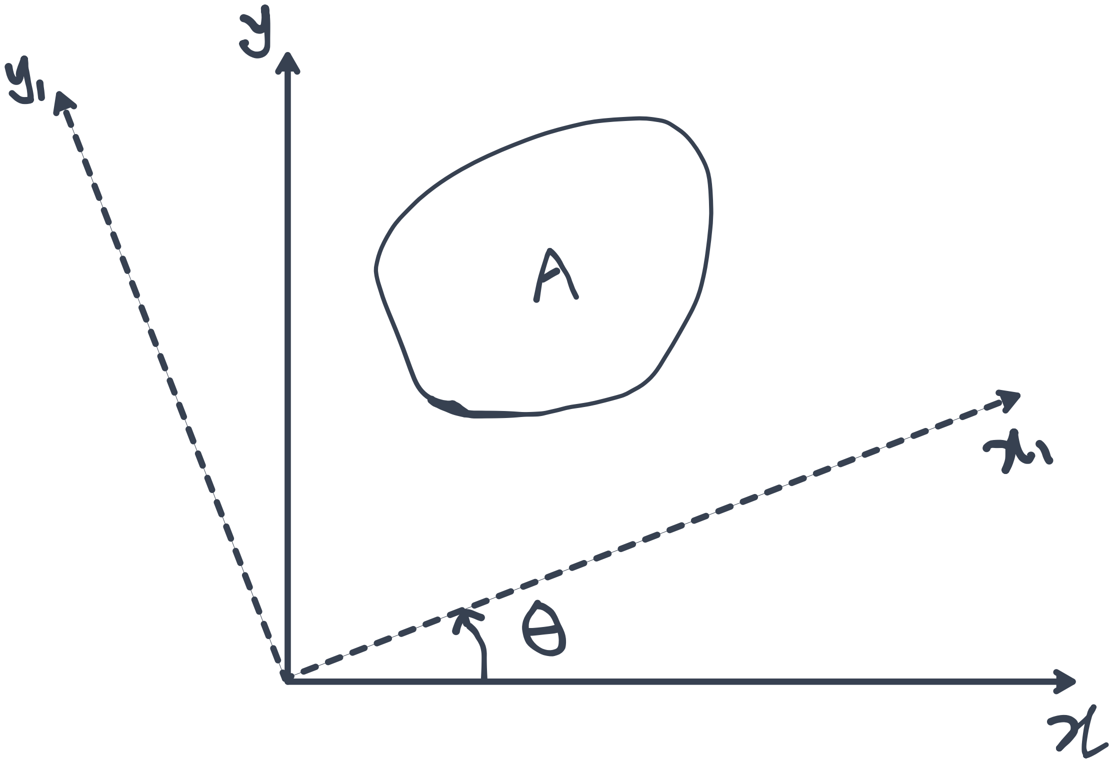
x, y, z are a set of axes. m, n, z are another set of axes.

If I_{xx} is at maximum, I_{yy} will be at minimum.

Transformation Law

The 2 sets of axes must share the origin.

> **Note**
>
> Don't have to memorize this. Will be given on exams, if required.



$$I_{x_1x_1} = \frac{I_{xx}+I_{yy}}{2} + \left(\frac{I_{xx}-I_{yy}}{2}\right)\cos 2\theta - I_{xy}\sin 2\theta$$

$$I_{y_1y_1} = \frac{I_{xx}+I_{yy}}{2} - \left(\frac{I_{xx}-I_{yy}}{2}\right)\cos 2\theta + I_{xy}\sin 2\theta$$

$$I_{x_1y_1} = \left(\frac{I_{xx}-I_{yy}}{2}\right)\sin 2\theta + I_{xy}\cos 2\theta$$

Principal Axes

The product of moment of area is 0 about principal axes.

$$I_{xy} = 0$$

There will be 2 directions of principal axes which are perpendicular to each other.

> ⓘ **Note**
>
> For a shape with more than 2 axis of symmetry, all axes through the centroid is a principal axis.

Principal second moments of area

Second moments about the principal axes.

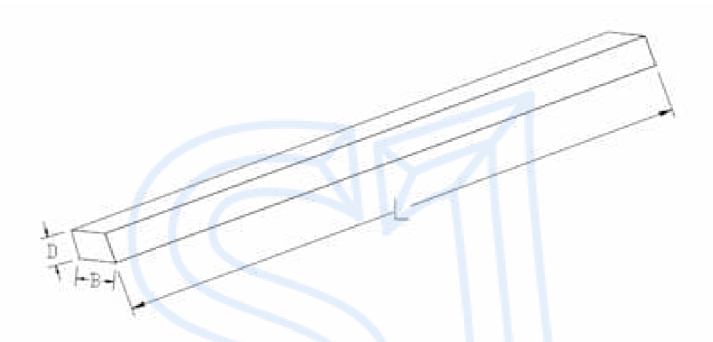
About principal axes second moments of area will be at minimum and maximum.

Centroidal principal axes

Principal axes through the centroid.

> **Note**
>
> Any axis of symmetry is a centroidal principal axis.

Beams

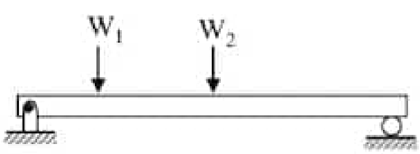


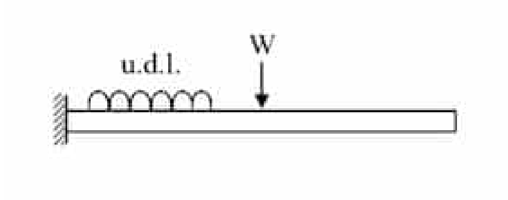
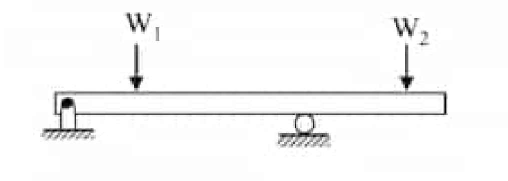
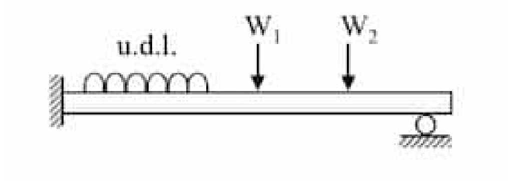
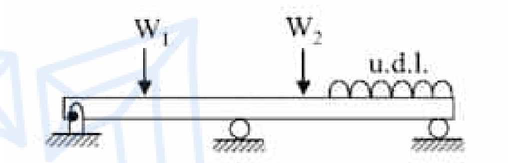
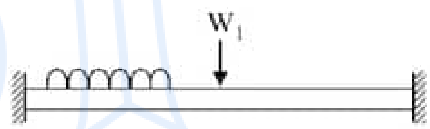
- long ($L >> B, D$)
- axis of the beam is straight
- constant cross-section throughout its length

Classified by supporting conditions

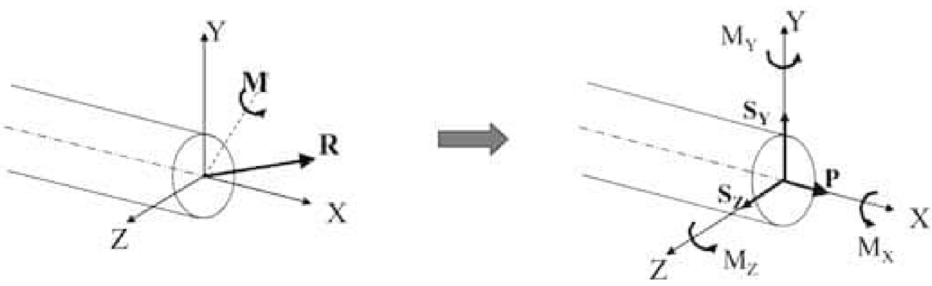
First 3 are the mandatory ones in s1.

u.d.l means uniformly distributed load.

Type	Image
Simply supported beam	

Type	Image
Cantilevered beam	
Overhanging beam	
Propped cantilevered beam	
Continuous beam	
Fixed beam	

At a section



- P - Normal force / Axial force
- S_y, S_y - Shear forces along y and z axis
- M_x - Twisting moment / Torque
- M_y, M_z - Bending moments about y and z axis

Degress of freedom

A plane member have 3 degrees of freedom. Any of the 3 can be restrained.

- Displacement in x -direction
- Displacement in y -direction
- Rotation about z -direction

SFD & BMD

Sign convention

- Bending moment
  - Hogging (curves upwards in the middle) is **(+) ve**
  - Sagging (curves downwards in the middle) is **(-) ve**
- Shear force
  - Clockwise shear is **(+) ve**.
  - Counterclockwise shear is **(-) ve**.

Pure bending

A member is in pure bending when shear force is 0 and bending moment is a constant.

Point of Contraflexure

The point about which bending moment is 0, and changes its sign through the point.

Distributed load, shear force & bending moment

Suppose a beam is under a distributed load of $w = f(x)$ per unit length.

$$\frac{\mathrm{d}S}{\mathrm{d}x} = -w$$

$$\frac{\mathrm{d}M}{\mathrm{d}x} = -S \quad \wedge \quad \frac{\mathrm{d}^2 M}{\mathrm{d}x^2} = w$$

Deflection of a beam

Suppose a simply supported beam is applied a load of W at mid-span.

$$S_{\max} = \frac{WL}{4I} \quad \wedge \quad D_{\max} = \frac{WL^3}{48EI}$$

Here:

- $S_{\max}$ - Maximum stress
- $D_{\max}$ - Deflection
- W - Load
- L - Span length
- E - Young's modulus
- I - Second moment of cross-sectional area

Principle of Superposition

A beam with multiple loads can be split into multiple systems each with a single load. Reason for doing so is the ease of calculations.

Values will be the sum of each system's corresponding value.

Structural Elements

3 types:

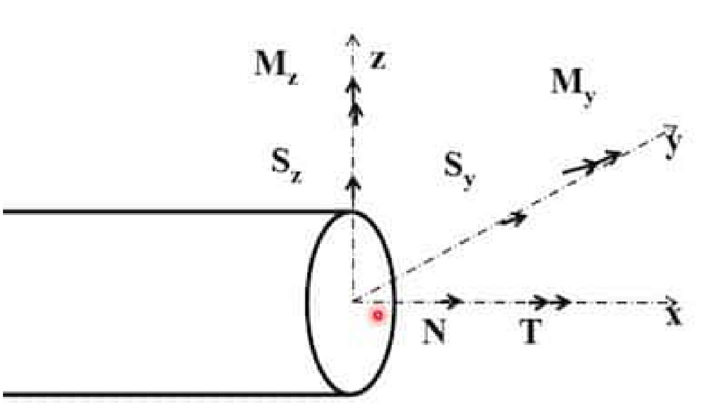
- Bars (1D)
- Plates and Shells (2D)
- Blocks (3D)

Bar elements are only focused for s1.

Pin Joint

Doesn't exert a moment. Free rotations are allowed. When only pin joints are used, bars will have only axial forces.

Bars



Here

- N - Axial force
- S_x, S_y - Shear force
- M_x

Types of bars

Axially loaded

Generally in trusses, **pin joints** are considered.

- Predominant tension - Ties
- Predominant compression - Struts

Flexural

- Predominant bending - beams

Torsional

- Predominant torque - shafts

Trusses

Also known as Ties-Struts model.

Definition

An assembly of members used to span long distances. Idealized as

- Connected by **frictionless** pin joints at their ends
- Developing axial forces

Types

2 types:

- Plane truss (2D)
- Space truss (3D)

A truss requires 3 external reactions for equilibrium.

Predominant force is axial force.

Advantages of truss

- High span
- Material efficiency

Triangulation

To create a truss:

- Start with a triangle (3 bars and 3 joints)
- Add 2 more bars and 1 joint repeatedly

This type of truss is a **simple truss**.

Simple (Closed) Truss

When a truss is only made of bars and joints.

Open Truss

When a truss is pinned directly to a foundation. It has 1 member & 2 free joints less than a closed truss.

Stability of trusses

When a truss is:

- unstable: it's called a mechanism
- stable: it's called a structure

Stable truss

When the shape cannot be altered, the structure is **internally stable**.

Stable & determinate (simply stiff)

Determinate means internal forces can be determined by laws of statics alone.

Stable & indeterminate

Indeterminate means laws of statics alone are not sufficient to determine forces; relative stiffness of members will influence the solution (Indeterminate trusses are safer).

Unstable truss

When the shape can be altered, the truss is called a mechanism.

Necessary condition for being simply stiff

> **Note**
>
> These are necessary (but not sufficient) conditions.

Here:

- m - Number of members (bars)
- j - Number of joints

For a 2D simple (closed) truss

- $m < 2j - 3$ - truss is unstable
- $m = 2j - 3$ - truss is determinate if stable
- $m > 2j - 3$ - truss is indeterminate if stable

For a 2D open truss

- $m < 2j$ - truss is unstable
- $m = 2j$ - truss is determinate if stable
- $m > 2j$ - truss is indeterminate if stable

For a 3D simple (closed) truss

$$m = 3j - 6$$

For a 3D open truss

$$m = 3j$$

Analysis of Trusses

Deviations from the ideal in real trusses.

- Loads are not applied only at joints; hence there is bending in members
- Joints are not perfectly pinned, so moments can be developed at joints

Method of Joints

Principle

Since the truss is in equilibrium, each pin joint must also be in equilibrium.

> **Note**
>
> 2 equilibrium equations can be written at each joint – vertical & horizontal.

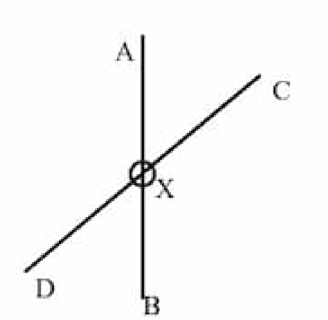
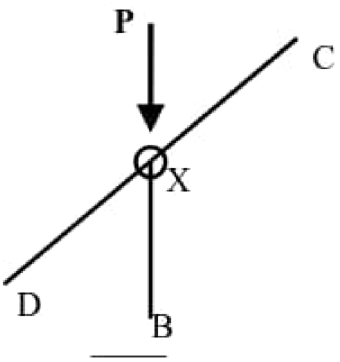
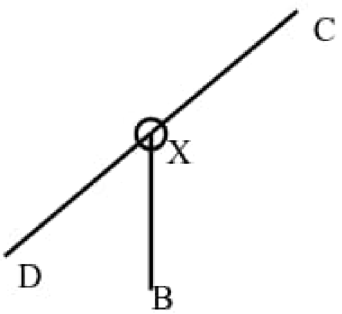
Sign convention

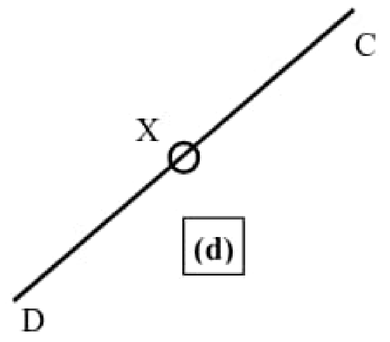
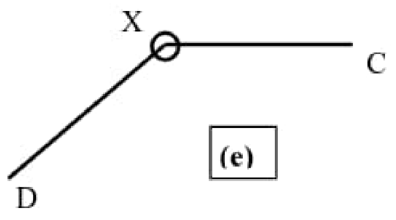
Forces acting on each joint is marked. Tensile forces are positive. Compressive forces are negative.

Method

- Find external reactions using equilibrium equations for the entire truss.
- Start with a joint with only 2 unknown joint forces.
- Mark the forces (consider all forces are tensile) acting on the joint.
- Find the unknown forces at the selected joint, using 2 equilibrium equations for the joint.
- Go to all other joints in turn and find forces in all the members.

Special cases

Case	Description
	$F_{\mathrm{AX}} = F_{\mathrm{XB}}, F_{\mathrm{DX}} = F_{\mathrm{XC}}$
	$F_{\mathrm{P}} = F_{\mathrm{XB}}, F_{\mathrm{DX}} = F_{\mathrm{XC}}$
	$F_{\mathrm{XB}} = 0, F_{\mathrm{DX}} = F_{XC}$

Case	Description
	$F_{\mathrm{DX}} = F_{\mathrm{XC}}$
	$F_{\mathrm{DX}} = F_{\mathrm{XC}} = 0$

Method of Sections

Principle

Since the truss is in equilibrium, each of its section must be in stable equilibrium.

Method

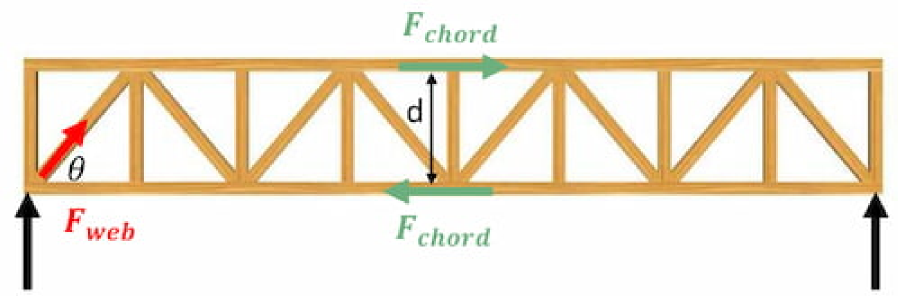
- Decide on which member's internal force must be calculated.
- Cut the truss through **3 or less** members including the target member.
- Internal forces in cut members become external forces. Can be represented as tensile forces.
- Use equilibrium equations for RHS or LHS section to find the internal forces.

Beam Analogy (Approximate) method

In this method, the internal forces are found assuming the elongated truss is a beam.

For a simply supported beam

- Maximum bending moment is at mid-span: $M_{\max} = \frac{wL^2}{8}$
- Maximum shear force is at the supports: $\frac{wL}{2}$



Here:

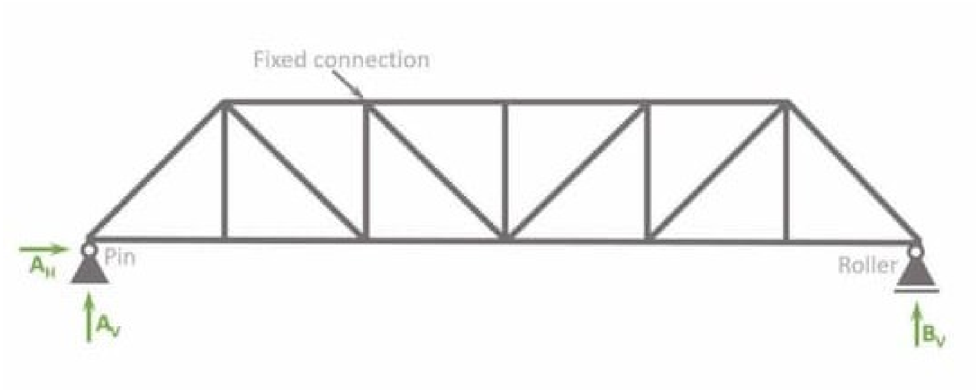
- Chord members - horizontal members
- Web members - diagonal members
- d - truss depth

In the truss,

- Bending moment is carried by chord members.
  $$\text{Bending moment} = F_{\text{chord}} \times d$$
- Shear force is carried by vertical component of web member force

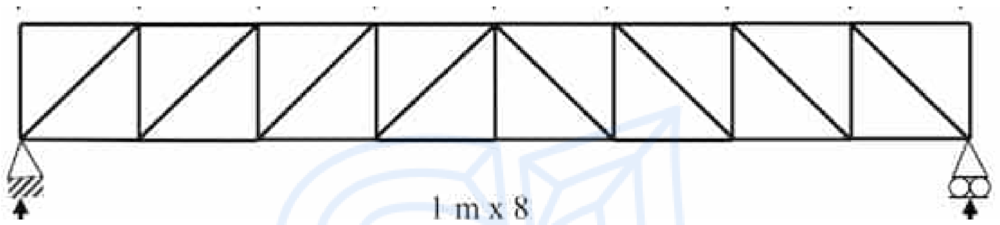
Pratt & Howe type trusses

Pratt type truss is shown below.



Internal force in web members are tensile.

Howe type truss is a shown below.



Internal force in web members are compressive.

Usually **Pratt type** is cost-efficient. To make sure a howe type truss is strong enough like pratt type, web members must be shorter and thicker.

Indeterminate Trusses

When a truss is indeterminate, one or more compatibility equations (related to truss deformation) must be used.